The Probability Distribution to Leptons and Quarks

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Abstract

The tracelike probability is expressed by the leptons and quarks Hamiltonians.

I use the following notations [1]:

$$I=\left[\begin{array}{cc}1 & 0\\ 0 & 1\end{array}\right],\ o=\left[\begin{array}{cc}0 & 0\\ 0 & 0\end{array}\right]$$

and the Pauli matrices:

$$\sigma_x=\left(\begin{array}{cc}0 & 1\\ 1 & 0\end{array}\right),\sigma_y=\left(\begin{array}{cc}0 & -i\\ i & 0\end{array}\right),\sigma_z=\left(\begin{array}{cc}1 & 0\\ 0 & -1\end{array}\right);$$

three chromatic Clifford's pentads:
the red pentad ζ:

$$\zeta^x=\left[\begin{array}{cc}\sigma_x & o\\ o & -\sigma_x\end{array}\right],\zeta^y=\left[\begin{array}{cc}\sigma_y & o\\ o & \sigma_y\end{array}\right],\zeta^z=\left[\begin{array}{cc}-\sigma_z & o\\ o & -\sigma_z\end{array}\right],$$

$$\gamma_\zeta^0=\left[\begin{array}{cc}o & -\sigma_x\\ -\sigma_x & o\end{array}\right],\ \zeta^4=-i\cdot\left[\begin{array}{cc}o & \sigma_x\\ -\sigma_x & o\end{array}\right];$$

the green pentad η:

$$\eta^x = \left[\begin{array}{cc} -\sigma_x & o \\ o & -\sigma_x \end{array} \right], \eta^y = \left[\begin{array}{cc} \sigma_y & o \\ o & -\sigma_y \end{array} \right], \eta^z = \left[\begin{array}{cc} \sigma_z & o \\ o & \sigma_z \end{array} \right],$$

$$\gamma_\eta^0 = \left[\begin{array}{cc} o & -\sigma_y \\ -\sigma_y & o \end{array} \right], \eta^4 = i \cdot \left[\begin{array}{cc} o & \sigma_y \\ -\sigma_y & o \end{array} \right];$$

the blue pentad θ:

$$\theta^x = \left[\begin{array}{cc} \sigma_x & o \\ o & \sigma_x \end{array} \right], \theta^y = \left[\begin{array}{cc} -\sigma_y & o \\ o & -\sigma_y \end{array} \right], \theta^z = \left[\begin{array}{cc} \sigma_z & o \\ o & -\sigma_z \end{array} \right],$$

$$\gamma_\theta^0 = \left[\begin{array}{cc} o & -\sigma_z \\ -\sigma_z & o \end{array} \right], \theta^4 = -i \cdot \left[\begin{array}{cc} o & \sigma_z \\ -\sigma_z & o \end{array} \right];$$

the light pentad β:

$$\beta^x = \left[\begin{array}{cc} \sigma_x & o \\ o & -\sigma_x \end{array} \right], \beta^y = \left[\begin{array}{cc} \sigma_y & o \\ o & -\sigma_y \end{array} \right], \beta^z = \left[\begin{array}{cc} \sigma_z & o \\ o & -\sigma_z \end{array} \right],$$

$$\gamma^0 = \left[\begin{array}{cc} o & I \\ I & o \end{array} \right], \beta^4 = i \cdot \left[\begin{array}{cc} o & I \\ -I & o \end{array} \right].$$

Hence:

$$\begin{array}{c} \beta^x = 0.5 \cdot (\beta^x + \zeta^x + \eta^x + \theta^x), \\ \beta^y = 0.5 \cdot (\beta^y + \zeta^y + \eta^y + \theta^y), \\ \beta^z = 0.5 \cdot (\beta^z + \zeta^z + \eta^z + \theta^z). \end{array}$$

Let

$$\langle \rho, j_x, j_y, j_z \rangle$$

be a probability current vector [2] and Ψ be any complex 4-spinor [1]:

$$\Psi = |\Psi| \cdot \left[\begin{array}{c} \exp(i \cdot g) \cdot \cos(b) \cdot \cos(a) \\ \exp(i \cdot d) \cdot \sin(b) \cdot \cos(a) \\ \exp(i \cdot f) \cdot \cos(v) \cdot \sin(a) \\ \exp(i \cdot q) \cdot \sin(v) \cdot \sin(a) \end{array} \right].$$

In this case the following system of equations

$$\left\{\begin{array}{c} \Psi^{\dagger}\cdot\Psi=\rho, \\ \Psi^{\dagger}\cdot\beta^{x}\cdot\Psi=j_{x}, \\ \Psi^{\dagger}\cdot\beta^{y}\cdot\Psi=j_{y}, \\ \Psi^{\dagger}\cdot\beta^{z}\cdot\Psi=j_{z} \end{array}\right|$$

has got the following type:

$$\left\{\begin{array}{c} \Psi^{\dagger}\cdot\Psi=\rho, \\ |\Psi|^{2}\cdot\left(\begin{array}{c} \cos^{2}(a)\cdot\sin(2\cdot b)\cdot\cos(d-g)- \\ -\sin^{2}(a)\cdot\sin(2\cdot v)\cdot\cos(q-f) \end{array}\right)=j_{x} \\ |\Psi|^{2}\cdot\left(\begin{array}{c} \cos^{2}(a)\cdot\sin(2\cdot b)\cdot\sin(d-g)- \\ -\sin^{2}(a)\cdot\sin(2\cdot v)\cdot\sin(q-f) \end{array}\right)=j_{y} \\ |\Psi|^{2}\cdot\left(\cos^{2}(a)\cdot\cos(2\cdot b)-\sin^{2}(a)\cdot\cos(2\cdot v)\right)=j_{z} \end{array}\right|.$$

Hence for every probability current vector: the spinor Ψ, obeyed to this system, exists.

The operator $\widehat{U}(t,\triangle t)$, which acts in the set of these spinors, is denoted as the evolution operator for the spinor $\Psi(t,\overrightarrow{x})$, if:

$$\Psi(t+\triangle t,\overrightarrow{x})=\widehat{U}(t,\triangle t)\,\Psi(t,\overrightarrow{x}).$$

$\widehat{U}(t,\triangle t)$ is a linear operator.

The set of the spinors, for which $\widehat{U}(t,\triangle t)$ is the evolution operator, is denoted as the operator $\widehat{U}(t,\triangle t)$ space.

The operator space is the linear space.

Let for an infinitesimal $\triangle t$:

$$\widehat{U}(t,\triangle t)=1+\triangle t\cdot i\cdot\widehat{H}(t).$$

Hence for an elements of the operator $\widehat{U}(t,\triangle t)$ space:

$$i\cdot\widehat{H}=\partial_{t}.$$

Since the functions ρ, j_x, j_y, j_z fulfill to the continuity equation [2]:

$$\partial_{t}\rho+\partial_{x}j_{x}+\partial_{y}j_{y}+\partial_{z}j_{z}=0$$

then:

$$\left(\left(\partial_t \Psi^\dagger\right) + \left(\partial_x \Psi^\dagger\right) \cdot \beta_x + \left(\partial_y \Psi^\dagger\right) \cdot \beta_y + \left(\partial_z \Psi^\dagger\right) \cdot \beta_z\right) \cdot \Psi =$$

$$= -\Psi^\dagger \cdot \left(\left(\partial_t + \beta_x \cdot \partial_x + \beta_y \cdot \partial_y + \beta_z \cdot \partial_z\right) \Psi\right).$$

Let:

$$\widehat{Q} = \left(i \cdot \widehat{H} + \beta_x \cdot \partial_x + \beta_y \cdot \partial_y + \beta_z \cdot \partial_z\right).$$

Hence:

$$\Psi^\dagger \cdot \widehat{Q}^\dagger \cdot \Psi = -\Psi^\dagger \cdot \widehat{Q} \cdot \Psi.$$

Therefore $i \cdot \widehat{Q}$ is the Hermitean operator.
Therefore:

$$\widehat{H} = \beta_x \cdot (i \cdot \partial_x) + \beta_y \cdot (i \cdot \partial_y) + \beta_z \cdot (i \cdot \partial_z) - i \cdot \widehat{Q}.$$

Let

$$-i \cdot \widehat{Q} =$$

$$\left[\begin{array}{cccc}
\varphi_{1,1} & \varphi_{1,2} + i \cdot \varpi_{1,2} & \varphi_{1,3} + i \cdot \varpi_{1,3} & \varphi_{1,4} + i \cdot \varpi_{1,4} \\
\varphi_{1,2} - i \cdot \varpi_{1,2} & \varphi_{2,2} & \varphi_{2,3} + i \cdot \varpi_{2,3} & \varphi_{2,4} + i \cdot \varpi_{2,4} \\
\varphi_{1,3} - i \cdot \varpi_{1,3} & \varphi_{2,3} - i \cdot \varpi_{2,3} & \varphi_{3,3} & \varphi_{3,4} + i \cdot \varpi_{3,4} \\
\varphi_{1,4} - i \cdot \varpi_{1,4} & \varphi_{2,4} - i \cdot \varpi_{2,4} & \varphi_{3,4} - i \cdot \varpi_{3,4} & \varphi_{4,4}
\end{array}\right],$$

here all $\varphi_{i,j}$ and $\varpi_{i,j}$ are a real functions on R^{3+1}.
Let:

$$\left\{\begin{array}{c} B_0 - B_z = \varphi_{3,3} \\ B_0 + B_z = \varphi_{4,4} \end{array}\right|,$$

$$\begin{array}{c} B_x = \varphi_{3,4}, \\ B_y = \varpi_{3,4}, \end{array}$$

$$\left\{\begin{array}{c} G + W_0 = \varphi_{1,1} - \varphi_{4,4} \\ G - W_0 = \varphi_{2,2} - \varphi_{3,3} \end{array}\right|,$$

$$W_1 = \varphi_{1,2} - \varphi_{3,4},$$
$$W_2 = -\varpi_{1,2} + \varpi_{3,4},$$

$$\left\{ \begin{array}{c} -a_\theta + a_\beta = \varphi_{1,3} \\ a_\theta + a_\beta = \varphi_{2,4} \end{array} \right|$$

$$\left\{ \begin{array}{c} b_\beta - b_\theta = \varpi_{1,3} \\ b_\beta + b_\theta = \varpi_{2,4} \end{array} \right|$$

$$\cos\left(\alpha_\theta\right) = \frac{a_\theta}{\sqrt{a_\theta^2 + b_\theta^2}},\ \sin\left(\alpha_\theta\right) = \frac{b_\theta}{\sqrt{a_\theta^2 + b_\theta^2}}$$

$$\cos\left(\alpha_\beta\right) = \frac{a_\beta}{\sqrt{a_\beta^2 + b_\beta^2}},\ \sin\left(\alpha_\beta\right) = \frac{b_\beta}{\sqrt{a_\beta^2 + b_\beta^2}}$$

$$m_\theta = 2 \cdot \sqrt{a_\theta^2 + b_\theta^2},\ m_\beta = 2 \cdot \sqrt{a_\beta^2 + b_\beta^2}$$

$$\gamma_\theta = \left(\cos\left(\alpha_\theta\right) \cdot \gamma_\theta^0 + \sin\left(\alpha_\theta\right) \cdot \theta^4\right)$$
$$\gamma_\beta = \left(\cos\left(\alpha_\beta\right) \cdot \gamma_\beta^0 + \sin\left(\alpha_\beta\right) \cdot \beta^4\right)$$

(here: for $\mu \in \{x, y, z\}$:

$$\gamma_\theta \cdot \theta^\mu = -\theta^\mu \cdot \gamma_\theta \text{ and}$$
$$\gamma_\theta \cdot \gamma_\theta = 1_4;$$

$$\gamma_\beta \cdot \beta^\mu = -\beta^\mu \cdot \gamma_\beta \text{ and}$$
$$\gamma_\beta \cdot \gamma_\beta = 1_4;$$

see [3])

$$\left\{ \begin{array}{c} -a_\zeta + b_\eta = \varphi_{1,4} \\ -a_\zeta - b_\eta = \varphi_{2,3} \end{array} \right|$$

$$\left\{\begin{array}{c} a_\eta - b_\zeta = \varpi_{1,4} \\ -a_\eta - b_\zeta = \varpi_{2,3} \end{array}\right|$$

$$\cos\left(\alpha_\zeta\right) = \frac{a_\zeta}{\sqrt{a_\zeta^2 + b_\zeta^2}},\ \sin\left(\alpha_\zeta\right) = \frac{b_\zeta}{\sqrt{a_\zeta^2 + b_\zeta^2}}$$

$$\cos\left(\alpha_\eta\right) = \frac{a_\eta}{\sqrt{a_\eta^2 + b_\eta^2}},\ \sin\left(\alpha_\eta\right) = \frac{b_\eta}{\sqrt{a_\eta^2 + b_\eta^2}}$$

$$m_\zeta = 2 \cdot \sqrt{a_\zeta^2 + b_\zeta^2},\ m_\eta = 2 \cdot \sqrt{a_\eta^2 + b_\eta^2}$$

$$\begin{array}{c} \gamma_\zeta = \left(\cos\left(\alpha_\zeta\right) \cdot \gamma_\zeta^0 + \sin\left(\alpha_\zeta\right) \cdot \zeta^4\right) \\ \gamma_\eta = \left(\cos\left(\alpha_\eta\right) \cdot \gamma_\eta^0 + \sin\left(\alpha_\eta\right) \cdot \eta^4\right) \end{array}$$

(here: for $\mu \in \{x, y, z\}$:

$$\begin{array}{c} \gamma_\zeta \cdot \zeta^\mu = -\zeta^\mu \cdot \gamma_\zeta \text{ and} \\ \gamma_\zeta \cdot \gamma_\zeta = 1_4; \end{array}$$

$$\begin{array}{c} \gamma_\eta \cdot \eta^\mu = -\eta^\mu \cdot \gamma_\eta \text{ and} \\ \gamma_\eta \cdot \gamma_\eta = 1_4; \end{array}$$

)

In this case:

$$\widehat{H} =$$

$$= 0.5 \cdot \left(\begin{array}{c} i \cdot (\beta^x \cdot (\partial_x - i \cdot B_x) + \beta^y \cdot (\partial_y - i \cdot B_y) + \beta^x \cdot (\partial_z - i \cdot B_z)) + \\ +m_\beta \cdot \gamma_\beta \end{array}\right) +$$

$$+0.5 \cdot \left(\begin{array}{c} i \cdot (\zeta^x \cdot (\partial_x - i \cdot B_x) + \zeta^y \cdot (\partial_y - i \cdot B_y) + \zeta^x \cdot (\partial_z - i \cdot B_z)) + \\ +m_\zeta \cdot \gamma_\zeta \end{array}\right) +$$

$$+0.5 \cdot \left(\begin{array}{c} i \cdot (\theta^x \cdot (\partial_x - i \cdot B_x) + \theta^y \cdot (\partial_y - i \cdot B_y) + \theta^x \cdot (\partial_z - i \cdot B_z)) + \\ +m_\theta \cdot \gamma_\theta \end{array} \right) +$$

$$+0.5 \cdot \left(\begin{array}{c} i \cdot (\eta^x \cdot (\partial_x - i \cdot B_x) + \eta^y \cdot (\partial_y - i \cdot B_y) + \eta^x \cdot (\partial_z - i \cdot B_z)) + \\ +m_\eta \cdot \gamma_\eta \end{array} \right) +$$

$$+ \left[\begin{array}{cccc} W_0 & W_1 - i \cdot W_2 & 0 & 0 \\ W_1 + i \cdot W_2 & -W_0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] +$$

$$+ \left[\begin{array}{cccc} G & 0 & 0 & 0 \\ 0 & G & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] +$$

$$+ \left[\begin{array}{cccc} B_0 & 0 & 0 & 0 \\ 0 & B_0 & 0 & 0 \\ 0 & 0 & B_0 & 0 \\ 0 & 0 & 0 & B_0 \end{array} \right].$$

References

[1] *The lepton, quark and hadron currents.*
http://xxx.lanl.gov/abs/physics/9806007

[2] *The probability in the relativistic m+1 space-time.*
http://xxx.lanl.gov/abs/physics/9803035

[3] *The Leptons and Gauge Bosons Masses Without Higgs.*
http://xxx.lanl.gov/abs/hep-ph/9812339